Gravitational resonances in mimetic thick branes

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Based on the works:
Y. Zhong, Y. Zhong, Y. P. Zhang, and Y. X. Liu,
Thick branes with inner structure in mimetic gravity, EPJC 78 (2018) 45;

Y. Zhong, Y. P. Zhang, W. D. Guo, and Y. X. Liu,
Gravitational resonances in mimetic thick branes, to appear in JHEP.

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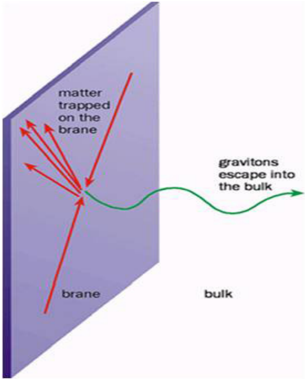
Outline

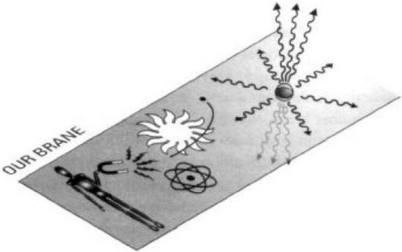
- Introduction
- Mimetic thick branes
- Gravitational resonances in mimetic thick branes
  - Gravitational resonances on a single brane
  - Gravitational resonances on the sub-branes
  - Gravitational resonances between the sub-branes
- Summary

Brief history of extra dimensions and braneworlds

- 1920's, Kaluza and Klein, KK Theory
- 1980's, Akama, Rubakov, Domain Wall Braneworld
- 1998, Arkani-Hamed, Dimopoulos, and Dvali, Large Extra Dimension (ADD Braneworld Scenario)
- 1999, Randall and Sundrum(RS), Warped Extra Dimension (RS thin Braneworld Scenario)
- 1999, DeWolfe, Freedman, Gubser, and Karch, Thick Braneworld Scenario

Picture of braneworlds





- 4D space-time is seen as a brane (hypersurface) embedded in higher-dimensional space-time
- Matter and gauge fields are confined on the brane, only the gravity can propagate in the bulk

Why gravitational resonances in mimetic thick branes

- Mimetic gravity gives a geometrical explanation of dark matter on galaxy level, cluster level and cosmological evolution and perturbation level [1501.02149, 1711.07290]
- It is also possible to unify the inflation and late-time acceleration period [1408.3561]
- Gravitational resonances can provide a new way to detect extra dimensions [PRL 84(2000)5932, EPJC75(2015)368]
- Mimetic thick branes can split into multi sub-branes [EPJC78(2018)45], and the effective potential of the tensor perturbation also splits into multi-wells, which may lead to new feature of gravitational resonances

The action of the 5D mimetic gravity is

$$S = \int d^5x\sqrt{-g}\left(\frac{R}{2} + \lambda\left[\partial^M\phi\partial_N\phi - U(\phi)\right] - V(\phi)\right), \tag{1}$$

where λ is a Lagrange multiplier.

Assuming the Minkowski brane metric

$$ds^2 = a^2(y)\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \tag{2}$$

the field equations read

$$\frac{3a'^2}{a^2} + \frac{3a''}{a} + V(\phi) + \lambda\left(U(\phi) - \phi'^2\right) = 0, \tag{3}$$

$$\frac{6a'^2}{a^2} + V(\phi) + 2\lambda\left(U(\phi) + \phi'^2\right) = 0, \tag{4}$$

$$\lambda\left(\frac{8a'\phi'}{a} + 2\phi'' + \frac{\partial U}{\partial\phi}\right) + 2\lambda'\phi' + \frac{\partial V}{\partial\phi} = 0, \tag{5}$$

$$\phi'^2 - U(\phi) = 0. \tag{6}$$

Redefining the extra-dimensional coordinate $dz = \frac{1}{a(y)}dy$, the perturbed metric in the new coordinate is given by

$$ds^2 = a(z)^2[(\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu + dz^2], \tag{7}$$

where the tensor perturbation $h_{\mu\nu} = h_{\mu\nu}(x^\mu, z)$ depends on all the coordinates and satisfies the transverse-traceless (TT) condition $\eta^{\mu\nu}\partial_\mu h_{\lambda\nu} = 0$ and $\eta^{\mu\nu}h_{\mu\nu} = 0$.

Next we redefine the perturbation as $h_{\mu\nu} = a(z)^{-\frac{3}{2}}\tilde{h}_{\mu\nu}$. After tedious but straightforward derivation, the perturbation equations are

$$\Box^{(4)}\tilde{h}_{\mu\nu} + \partial_z^2\tilde{h}_{\mu\nu} - \frac{\partial_z^2 a^{\frac{3}{2}}}{a^{\frac{3}{2}}}\tilde{h}_{\mu\nu} = 0. \tag{8}$$

Employing the decomposition $\tilde{h}_{\mu\nu} = \epsilon_{\mu\nu}(x^\gamma)\mathrm{e}^{ip_\lambda x^\lambda} t(z)$, we obtain the Schrödinger-like equation for the extra-dimensional part $t(z)$ of the tensor perturbation:

$$-\partial_z^2 t(z) + V_{\mathrm{t}}(z)t(z) = m^2 t(z), \tag{9}$$

where the effective potential $V_{\mathrm{t}}(z)$ is given by

$$V_{\mathrm{t}}(z) = \frac{\partial_z^2 a^{\frac{3}{2}}}{a^{\frac{3}{2}}}, \tag{10}$$

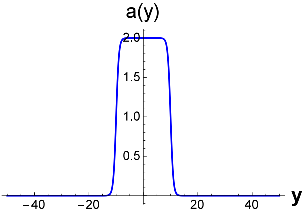
and m is the mass of the tensor perturbation $t(z)$. The zero mode of the tensor perturbation is

$$t_0(z) \propto a^{\frac{3}{2}}(z), \tag{11}$$

which is localized on the brane for our latter solutions.

Single brane model:

$$a(y) = \tanh[k(y+b)] - \tanh[k(y-b)]. \tag{12}$$



(a) warped factor (b) effective potential

Figure: The shapes of the warp factor $a(y)$ and the effective potentials $V_\mathrm{t}(z)$ for the single brane model (12) with $k=1$ and $b=10$.

It can be seen that the potential $V_\mathrm{t}(z)$ has an obvious double-well with two barriers, which is the main reason leading to resonance KK modes.

To solve the Schrödinger-like equation (9) for $t(z)$ numerically, we decompose $t(z)$ into an even parity mode $t_e(z)$ and an odd parity mode $t_o(z)$, which are set to satisfy the following boundary conditions:

$$t_e(0) = 1, \qquad \partial_z t_e(0) = 0; \tag{13}$$

$$t_o(0) = 0, \qquad \partial_z t_o(0) = 1. \tag{14}$$

To investigate the gravitational resonances, we adopt the concept of the relative probability of the KK mode $t(z)$ with mass m:

$$P(m^2) = \frac{\int_{-z_b}^{z_b} |t(z)|^2 dz}{\int_{-z_{max}}^{z_{max}} |t(z)|^2 dz}. \tag{15}$$

Here $2z_b$ is approximately the width of the thick brane, and $z_{max} = 10z_b$.

For a given m^2, the Schrödinger-like equation (9) can be solved numerically for the even parity mode $t_e(z)$ and the odd parity mode $t_o(z)$ with the conditions (13) and (14), respectively. Then the relative probability P corresponding to this $t_e(z)$ or $t_o(z)$ can be obtained. By this means, the relative probability as a function of m^2 is obtained and plotted in Fig. 2, in which each of the peaks represents a resonance mode.

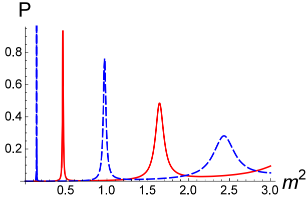


Figure: The relative probability $P(m^2)$ of the even parity mode t_e (red solid lines) and the odd parity mode $t_o(z)$ (blue dashed lines) for the single brane model (12). The parameters are set to $k = 1$ and $b = 10$.

Furthermore, the corresponding life-time τ of the resonances can be obtained by $\tau = \frac{1}{\Gamma}$, where Γ is the full width at half maximum (FWHM).

The resonances having large life-time can be quasi-localized on the brane for a long time. Therefore, these resonances are approximately four-dimensional gravitons.

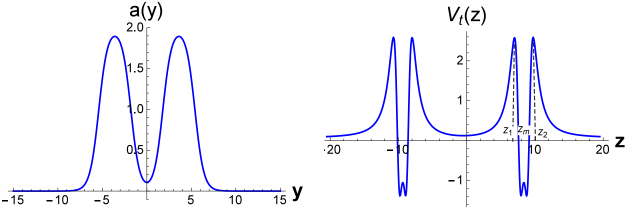
We find that the relative probability P and life-time τ of the resonance modes decrease with the mass square m^2, while the FWHM Γ increases with m^2. Thus, the behavior of the resonances in the single mimetic brane is similar to that in a single brane model in general relativity.

Gravitational resonances on the sub-branes

Double brane model:

$$\begin{aligned} a(y) &= \tanh[k(y+d+b)] - \tanh[k(y-d-b)] \\ &- \tanh[k(y+d)] + \tanh[k(y-d)]. \end{aligned} \quad (16)$$

where $2(b+d)$ is approximately the thickness of the brane, and $2d$ is the distance between the two sub-branes in the physical coordinate y.



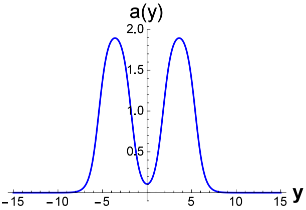
(a) warped factor (b) effective potential $V_{\rm t}(z)$

Gravitational resonances on the sub-branes

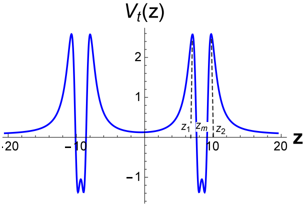
Relative probability P_3:

$$P_3 = \begin{cases} \dfrac{\int_{z_1}^{z_2} |t(z)|^2 dz}{\int_{z_m-5(z_2-z_1)}^{z_m+5(z_2-z_1)} |t(z)|^2 dz}, & z_m \geq 5(z_2 - z_1) \\[2ex] \dfrac{\int_{z_1}^{z_2} |t(z)|^2 dz}{\int_{0}^{10(z_2-z_1)} |t(z)|^2 dz}, & z_m < 5(z_2 - z_1) \end{cases} \tag{17}$$

where (z_1,z_2) is the z-coordinate range of one of the sub-wells.



(c) warped factor



(d) effective potential $V_{\mathrm{t}}(z)$

Gravitational resonances on the sub-branes

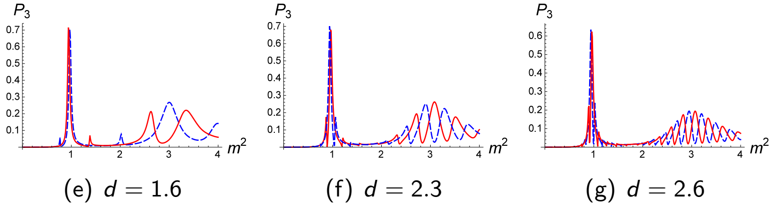


Figure: The relative probability $P_3(m^2)$ of the even parity mode $t_e(z)$ (red solid lines) and the odd parity mode $t_o(z)$ (blue dashed lines) quasi-localized on the sub-branes for the double brane model (16). The parameters are set to $k = 1$, and $b = 7$.

Gravitational resonances on the sub-branes

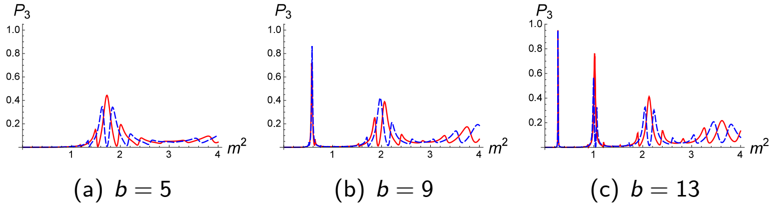


Figure: The relative probability $P_3(m^2)$ for the double brane model (16) with $k = 1$ and $d = 2.3$.

Gravitational resonances on the sub-branes

- The relative probability of the resonance modes do not monotonically decrease with the mass square m^2, and there are only one odd and one even significant resonance modes, which is different from the case in single brane model. .
- As the brane distance d increases, more resonances appear.
- As the sub-brane thickness increases, the mass of the first even and odd modes decrease, while their relative probability increase.
- For small sub-brane thickness $b = 5$, there are only a group of resonances with small relative probability, while for large sub-brane thickness, the resonances with large relative probability appear.

The warped factor $a(y)$ is also assumed as Eq. (16). The relative probability P_4 corresponding to the gravitational resonances quasi-localized between the two sub-branes is given by

$$P_4 = \frac{\int_{-z_1}^{z_1} |t(z)|^2 dz}{\int_{-10z_1}^{10z_1} |t(z)|^2 dz}, \tag{18}$$

where z_1 is shown in Fig. 3(d).

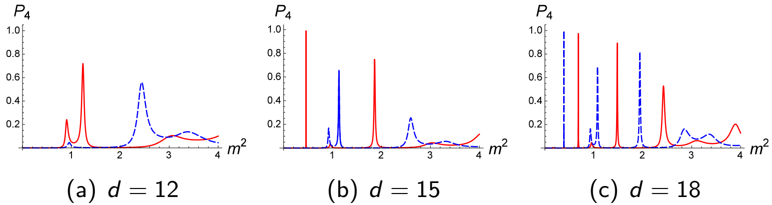
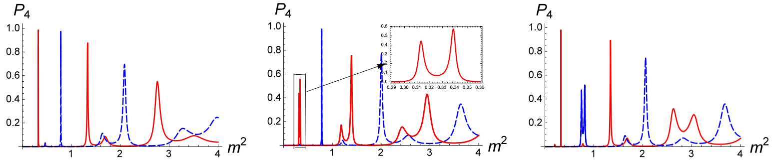


Figure: The relative probability $P_4(m^2)$ of the KK modes quasi-localized between the sub-branes for different values of the distance d. The parameter k is set to $k = 1$, $b = 7$, and $d = 12, 15, 18$.

- The number and life-time of the resonances increase with the width d of the middle sub-well, which is similar to the case of a single brane .
- While the relative probability of the resonances does not monotonically decrease with the mass square m^2, which is very different from the case of a single brane



(a) $b = 11$ (b) $b = 11.9$ (c) $b = 14.9$

Figure: The relative probability $P_4(m^2)$ of the KK modes quasi-localized between the sub-branes for different values of the brane thickness b. The parameters are set to $k = 1$, $d = 2.3$, and $b = 11, 11.9, 14.9$.

- Though the parameter b is related to the sub-wells on the left and right rather than the one in the middle, it has an important impact on the resonance quasi-localized between the sub-branes.

Summary

- In the single brane model, the gravitational resonances is similar to the case in general relativity.
- In the single brane model, the influence of the brane structure are analyzed, and new feature were found for the gravitational resonances quasi-localized on sub-branes and between the sub-branes.

Thanks!